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A model calculating machine for a very general type of non-linear differential equation and its application as a dynamic electron-path recorder.

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1. The original task

The automatic electron-path recorder plays an increasingly important part in modern electronics. The problems of electron-optics assume an ever increasing importance and are also becoming continuously more complicated. But also other processes, such as the movement of an electron in the modern magnetron, have to be dealt with from the mathematical standpoint. The mathematical, or even only the graphic, solution of such problems is a very voluminous and time-consuming task. Therefore, a model calculating machine for the solution of such problems, called an electron-path-recorder, is of great importance in the development of electron tubes.

Realizing the importance of the problem, it is a matter of course that experiments aimed at solving it have been initiated at an early stage. The oldest solution [1], like most graphic methods, is based on the utilization of a simple formal relationship between (electron)-path curvature and intensity of the electric field. Figuratively speaking, this method is an automatic control of a graphic process of electron-path construction. According to this principle, it is possible to record the paths in that type of electrostatic fields which can be represented by models in an electrolytic tank.

The operative principle just described still has a few serious deficiencies, however. One succeeds only in plotting the path-curves, but not their time slope. Basically, it would be possible to consider also the time factor without increasing the number of coordinates of the plot by indicating time scales along the path. However, the time function does not appear explicitly during the entire calculation process because the operative principle is not based on the dynamic basic equations of the electron movement. Therefore, the plot is limited to electrostatic fields. But even in electrostatic fields the time slope may already play a role, as in the case of drift spaces in which time-focusing effects are to be studied.

In modern electronics, fields which are variable with time play a role almost equally important with that of static fields. The processes in diodes and triodes in the centimeter-wave range belong in this field. Problems of this nature are not solvable according to this old method. An even more general problem is the significance of the magnetic field in the electron movement in the magnetron. In such a case, a magnetic field is present perpendicular to the plane of motion. The electric field is periodically variable. This sort of problem is still relatively simple and promising, inasmuch as the motion of the electron still occurs in a plane in spite of the presence of a magnetic field. In comparison to this problem, the consideration of a so-called guide-field in cathode-ray tubes (thermionic tubes) or of a magnetic lens promises little success.

The new automatic electron-path-recorder is designed to solve also the just described expanded problem, namely, the plotting of the time slope on every electron-path, and the possibility of taking into account the time variations of the fields and of a magnetic field vertical to the plane of the path. A new principle was developed which is directly based on the dynamic basic equations of the electron motion; consequently, the device is a dynamic model.

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1) David B. Langmuir: Nature Nr. 139, Page 1066, 1937 and R.C.A. Review vol. XI March 1950 Nr. 1, Page 143

During the solution of the problem an apparatus came into being which is a valuable model calculating machine both in regard to its basic principle and to its many technical details. It has a general significance reaching far beyond the special task of plotting electron motion, because it may be built up into a general-purpose model calculator. Therefore, it appears reasonable and appropriate to choose a description which demonstrates, first of all, the general calculating principle and its application to general differential equations beyond a limited scope, and only in the second place to discuss the special application of the machine as dynamic electron-path-recorder. This manner of proceeding has the additional advantage that the most important features of the apparatus are explained in a particularly understandable outline form.

II. The general mathematical problem

The motion of a mass point in a plane is represented in the most general manner by the following system of equations:

1) $$F(x, t, \dot{x}, \ddot{x}, \dot{y}, \ddot{y}) = 0$$

$$G(x, t, \dot{x}, \ddot{x}, \dot{y}, \ddot{y}) = 0$$

In this system, the functions F and G may have a very general and also a non-linear character. For the motion of a mass point in space, the corresponding system applies as follows:

2) $$F(x, t, \dot{x}, \ddot{x}, \dot{y}, \ddot{y}, \dot{z}, \ddot{z}) = 0$$

$$G(y, t, \dot{x}, \ddot{x}, \dot{y}, \ddot{y}, \dot{z}, \ddot{z}) = 0$$

$$H(z, t, \dot{x}, \ddot{x}, \dot{y}, \ddot{y}, \dot{z}, \ddot{z}) = 0$$

In both systems with still more degrees of freedom, corresponding even more extensive systems of equations may apply.

In the following, a model calculating machine is described, which permits the solution of the system of equations 1 within a relatively short time (and in a correspondingly longer period of time also 2) if it may be reduced to the specific form:

3) $$f_0 = f_1 \bigcirc f_2 \bigcirc \cdots\cdots \bigcirc f_{15}$$

$$g_0 = g_1 \bigcirc g_2 \bigcirc \cdots\cdots \bigcirc g_{15}$$

whereby f_0 and g_0 are functions of $\ddot{x}$ and $\ddot{y}$, representable by unidimensional and two-dimensional, scannable models, of any of the magnitudes $t, x, \dot{x}, y, \dot{y}$,
and $f_1 \ldots\ldots f_{15}$, $g_1 \ldots\ldots g_{15}$ are functions, representable by unidimensional and two-dimensional, scannable models, of two each of the magnitudes $t, x, \dot{x}, y, \dot{y}$; that is, of

4)

t,t	t,x	$t,\dot{x}$	t,y	$t,\dot{y}$
	x,x	$x,\dot{x}$	x,y	$x,\dot{y}$
		$\dot{x},\dot{x}$	$\dot{x},y$	$\dot{x},\dot{y}$
			y,y	$y,\dot{y}$
				$\dot{y},\dot{y}$

whereby the terms of the diagonal of the incomplete matrix lead to unidimensional, the other terms to two-dimensional models, and whereby the symbol $\bigcirc$ represents a mathematical operation between the individual functions which may be executed by an electrical or electromechanical model-calculating operation.

Among the mathematical operations which may be executed electrically are particularly the four basic arithmetic operations: addition, subtraction, multiplication, and division. An additional arithmetically frequently useful operation, although it does not offer any innovations from the mathematical viewpoint, is feasible, namely, the multiplication of the left sides with a function of one of the pairs of values of the table 4.

All the important features of the calculating machine may already be recognized by considering a differential equation of the 2nd degree of the form of

5). $$f_0 = f_1 \bigcirc f_2 \bigcirc \dots \bigcirc f_6,$$

whereby f_0 is a function of $\ddot{x}$ and of one of the magnitudes among $t, x, \dot{x}$, each, and whereby $f_1 \dots f_6$ are functions of two magnitudes among $t, x, \dot{x}$, each, such as of

6)	t,t	t,x	$t,\dot{x}$
		x,x	$x,\dot{x}$
			$\dot{x},\dot{x}$

A system having several simultaneous differential equations does not generally furnish any new aspects. Only the dynamic electron-path-recorder offers two additional aspects, namely, the facts that a two-dimensional electrolytic tank of the type which is commonly used for the determination of potentials is utilized instead of a plurality of scannable models, and that the end result is not given in the form of single functions of time, but in a plane path diagram with time scale.

III. The electromechanical basic unit of the calculating machine, the integrator aggregate

An electromechanical device for a double integration of time, the integrator aggregate, plays an essential part in the operation of the model calculating machine. In order to exemplify its mode of operation very clearly, the discussion will be limited at first to the case

7) $$f_0 = \ddot{x}.$$

The solution of equation (5) will be discussed at the end of the following chapter. The equation which has to be solved primarily has the following form

8) $$\ddot{x} = f_1 \bigcirc f_2 \dots \bigcirc f_6.$$

A good model for the solution of this equation is a flywheel mass driven by an electric motor. Providing the angle of rotation is assigned the symbol φ, the moment of inertia of the entire flywheel arrangement bears the symbol Θ, and the moment of rotation is represented by D, the following equation results for the device

9). $$\ddot{\varphi} = \frac{D}{\Theta}$$

The moment of rotation consists of two parts, the deflecting (interfering) moment of rotation, $D_{stör}$, and the required moment of rotation, D_{soll}, thus

10) $$\ddot{\varphi} \quad \frac{D_{stör}}{\Theta} + \frac{D_{soll}}{\Theta}$$

In order to solve equation (8) by the given device, x is to be represented by φ. If one proposes to plot the right side of equation (8) against the second term of the right side of equation (10), the first term must be neutralized. This is best accomplished by reducing it (the first term) to an immeasurably small quantity against the second term. We shall first discuss this problem.

The deflecting moment of rotation (störende Drehmoment) which entails a retarding effect has the following composite parts:

1. A retarding (brake coupling) moment corresponding to the momentum (propulsion requirement) of the indicator.

2. A retarding moment corresponding to the momentum (propulsion requirement) of certain auxiliary devices (scanning devices) for the realization of the function $f_1, f_2 \ldots\ldots f_6$.

3. The frictional retarding moment.

The two first parts (of the retarding effect) may be eliminated by a single measure, namely, through introduction of a servomechanism. The third part of the retarding moment may be reduced considerably below the original value by rotating the main sources of the frictional retarding moment, the bearings, by means of a servomechanism at an angle nearly identical to the flywheel mass.

Servomechanisms for the transmission of an angle of rotation without imposing any change on the steering organ, the so-called chargeless control amplifiers of the moment of rotation, may be constructed on various principles. The most essential element of such a transmission is the discriminator, which continuously measures the shift angle between steering and steered axis, deriving from the shift angle the steering tension for the auxiliary motor which furnishes the amplifying moment of rotation (servomotor). The device must be built in such a manner that the shift mistake tends to reduce itself. This tendency is achieved by using an electromechanical compact loop with impedance coupling effect. In the case under review, a rotation plate condenser is utilized as discriminator. Steering and steered axis have a common revolution line. The rotor is connected with the steering axis, and the stator of the rotating-plate condenser is connected with the steered axis. The rotating-plate condenser is switched into an alternating current (measuring) bridge which indicates a value 0 at a medium (rotary) displacement of the condenser. If a shift angle occurs, a tension (voltage) is created at the bridge, which is quantitatively proportional to the phase angle (shift angle) and whose phase, against the bridge-supply voltage, indicates the sense of direction of the shift angle. From such a reading, a steering voltage for the servomotor may easily be derived. The motor propels the indicator and the auxiliary devices necessary for the realization of functions $f_1, f_2 \ldots\ldots f_6$.

The reduction of the retarding moment is most easily accomplished in the following manner: a vertical axle is used for the flywheel mass and the primary driving motor (hereafter called integrator). Thus the friction is essentially localized to the step-bearing. By propelling the step-bearing from the servomotor, it is possible to greatly reduce friction because there remains, then, only a relative motion between the axle and the bearing corresponding to the shift angle. This principle was applied to the constructed (experimental) model. A further improvement could be obtained by completely omitting the step-bearing, and by supporting the rotating part by a thread, suspended in one of the housings driven by the servomotor. Inasmuch as the relative motion of the flywheel mass and housing is equal to the shift angle which can only be increased to a certain value (45° for instance),

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such a suspension is possible without bearing. The resiliency of the supporting thread (thin steel wire) may be kept at a negligible minimum.

An indication as to how far one has to go quantitatively speaking in the elimination of the retarding (brake) effect may be obtained by the following reflections: if the right side of the equation retains the value 0 for a long time during an arithmetical operation, the previously attained velocity may only decrease within the bounds of the admissible inaccuracy If one is to remain on the safe side, he must require that the velocity decrease throughout the total time interval of an arithmetical operation remains smaller than the admissible inaccuracy value. If the latter is one per cent, for instance, the time constant of the aggregate has to be one hundred times as long as the time interval of the arithmetical operation.

After elimination of the deflecting moment of rotation, there remains a last essential problem whose solution will secure a faultless operation of the integrator aggregate. The term $\underline{D_{soll}}$, e.g., the right side of the equation (6) will appear in the form of a voltage. Therefore, it is essential to make sure that also the mechanical moment of rotation of the integrator motor be proportional to this voltage on the entire working scope. This requirement is fulfilled by the selection of an integrator motor with appropriate characteristics. Particularly suitable are two-phase induction motors whose first phase is supplied with a constant voltage, whereas the second phase is switched to the driving voltage. Such a motor retains a strict proportionality between the amplitude of the voltage and the moment of rotation over a large range.

The multi-phase induction motor, on the other hand, has a disturbing source of error. Its moment of rotation depends on velocity $\dot{\varphi}$. A normal multi-phase motor has a moment of rotation (torque) / rotative speed-characteristicum as plotted in Illustration 1, Curve A. The moment of rotation increases at first with increasing rotative speed until it reaches the so-called breakdown-torque, and subsequently it decreases rapidly to reach 0 at the synchronized number of revolutions. For problems of this type, one builds motors whose breakdown-torque coincides with the rotative speed 0, as represented by curve B.

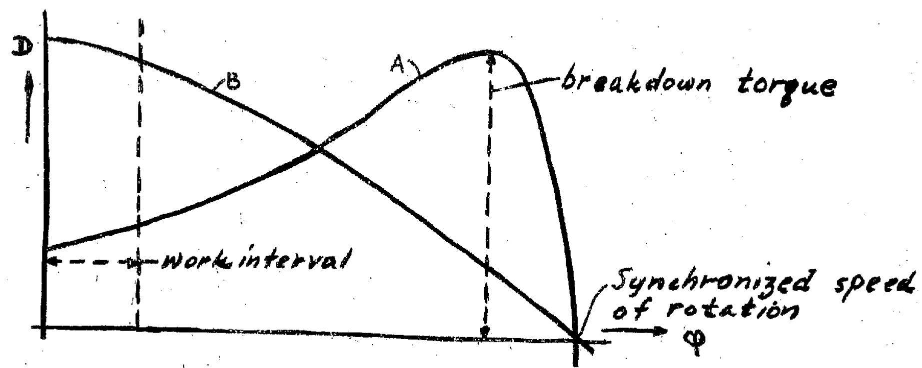

Illustration 1: Moment of rotation (torque)/rotative speed-characteristics of multi-phase induction motors.

All the two-phase induction motors developed for servo-purposes have this quality because it is also an essential requirement in the prevention or easy suppression of servo-vibrations. These motors have a moment of rotation (torque) which is practically independent of the rotative speed at low rotative values. Therefore, it is only necessary to impose appropriate limitations on the rotative speed. In order to obtain an efficient utilization of the servomotor despite this fact, a reduction gear is switched between the servomotor and the controlled axle with the increased torque.

The condensation of the ideas developed so far leads to the construction of an integrator aggregate for the solution of equation (8) according to the diagram, Illustration 2. Flywheel mass and rotor of the integrator are constructed rigidly as one unit. Also rigidly connected thereto is the inner axle which extends unilaterally upward. The inner axle is hollow in order to obtain appropriate space in its interior for as long a thread as possible bearing the weight of the flywheel mass. The inner axle is also hollow. The external axle is supported by the space-solid external bearings. The thread is suspended at the upper end of the external axle. The stator of the integrator motor is fixed in its place. The external axis also supports the housing. The discriminator-condenser is located between the flywheel mass and the housing. It consists of a cylindrical arrangement with 90° segments, two segments facing each other for the purpose of balancing the masses. The "rotor" is fixed directly to the flywheel mass; the "stator" is isolated in the housing. The isolated section is connected by a conductor to the sliding (contact) ring mounted outside the housing from where a connection leads to the electrical servo-section by way of a sliding contact. Another sliding contact connects the external axle and the "rotor" fixed to the flywheel mass with the electrical servo-section via the conducting (suspension) thread.

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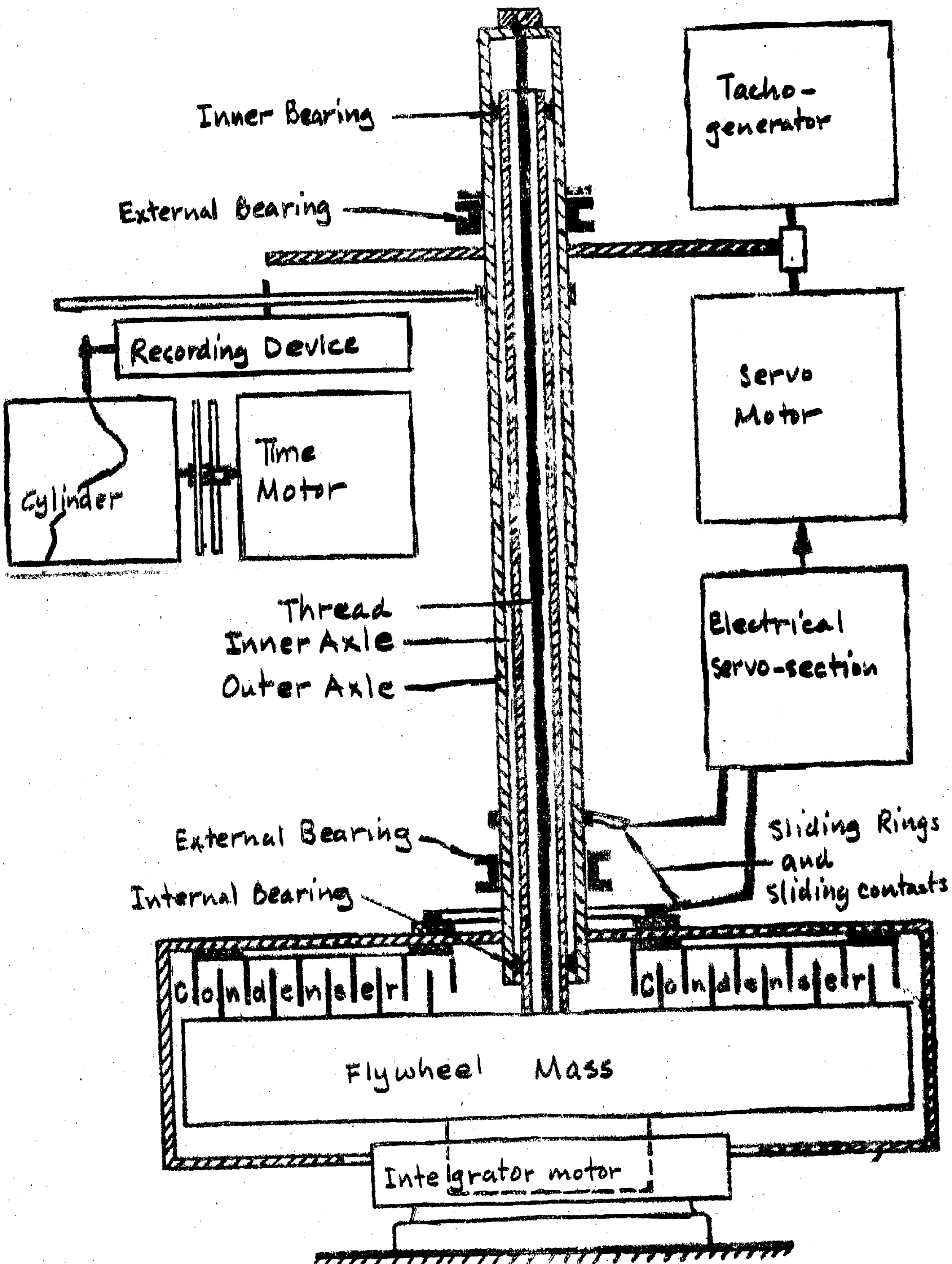

Illustration 2 : Diagram of Integrator Aggregate

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The electrical servo-section consists of bridge, bridge supply arrangement, amplifier, and possibly also a transformer element which adjusts the initial voltage to the requirements of the servomotor. The housing is built around the flywheel mass in order to reduce the air friction of the flywheel mass. Housing and rotor of the servomotor are to have as low moments of inertia as possible. This requirement is particularly applicable to the servomotor because it has a considerably higher number of revolutions (than the axle) as a consequence of the interposed reduction gear arrangement, and because the moments of inertia change (increase) to the square of the reduction ratio. Therefore, the most appropriate motor is a so-called "Bechermotor" (literally bucket-motor) with minimum inertia. The maintenance of the smallest possible moment of inertia of the above-mentioned parts is essential to the reduction of the shift angle (angular error) and to the avoidance of servo-vibrations.

A recording device is driven by the external axle via another reduction gear; this device transforms the rotary movement into a linear movement of the recorder style. The style registers on a cylinder which is driven by the "time motor" at a constant rate of speed. There must be readily serviceable coupling in the recording device in order to permit an easy adjustment of the initial magnitude. A tacho-generator is mounted together with the servomotor on the same axle. This tacho-generator is a special type of alternating current generator which furnishes an alternating voltage of equal frequency as the supply voltage of the integrator motor. The amplitude of the alternating voltage is proportional to the rotative speed. Also this tacho-generator is to have the bucket-design because its moment of inertia adds up directly with that of the servomotor.

IV. The solution of a non-linear differential equation of 2nd degree with single functions depending on one magnitude only.

To complete a model calculating device designed for the solution of equation (8), one needs to consider also the auxiliary devices necessary for the display of the right side (of the equation). Among the auxiliary devices are the scannable models, their single functions, and the devices executing the intermediary arithmetical operations. This entire complex may assume various forms and have diverse dimensions. It is therefore most appropriate to discuss it in detail in its application to special equations and to demonstrate the solutions of these equations. We shall begin with equations whose single functions depend upon only one of the magnitudes t, x, $\dot{x}$, and which may be displayed in a given case by one-dimensional models.

First the equation

11) $\ddot{x} = f_1(t)$

is to be considered. If there are no special time functions at hand which may easily be realized technically, one may transcribe the function $f_1(t)$ manually into a voltage function. For this purpose the following auxiliary device is appropriate: the function $f_1(t)$ plotted on a curve sheet is set up on a cylinder driven by a motor at a constant rate of speed (cf. Illustration 3). A style which may be used for plotting on the curve sheet, in parallel direction to the cylinder axis, is mechanically connected to the slide contact (adjusting slider) of a potentiometer 1. The potentiometer is supplied by means of a transformer with an alternating voltage whose frequency [800 Hz (Hertz) for instance] is suited for the propulsion of the integrator motor.

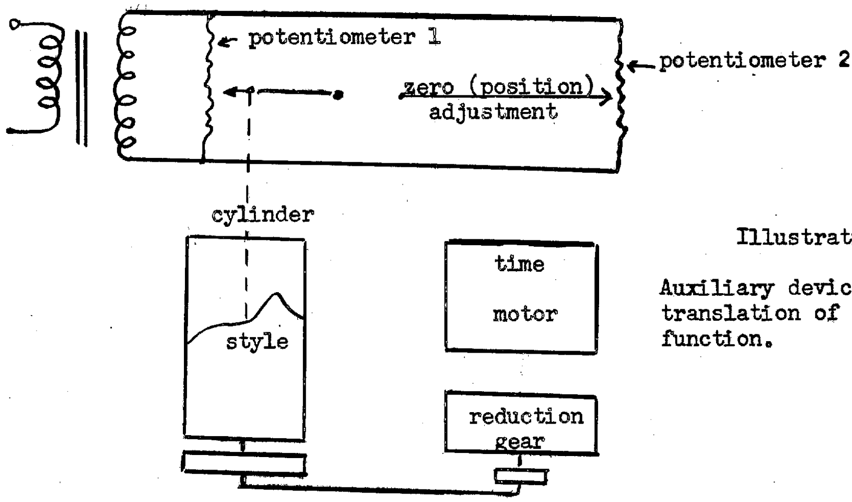

Illustration #3.

Auxiliary device for the translation of the time function.

Another potentiometer 2 with which the zero value of the function can be set (on the dial) in such a manner that the zero axis may lie on the paper at any point is located parallel to potentiometer 1. From the two contact slides, the voltage is conducted either directly or via interposed amplifier to the integrator motor. The second phase of the integrator motor is constantly supplied with a 90° de-phased (displaced in phase) voltage.

For the execution of the arithmetic operation one still needs to set the initial conditions. The coupling in the recording device is opened. The style is set on the original coordinate. Then the flywheel mass is pre-accelerated by means of an auxiliary voltage until it reaches the prescribed initial velocity; this stage is measurable by the voltage of the tacho-generator. At the moment of starting the curve scanning, the coupling in the recording device is closed, and the integrator motor is connected to the voltage originating from the contact slides. The arithmetical operation will then be well under way provided that the curve is being redrawn correctly as the operation progresses. Instead of a curve-sheet, a template may also be used whereby the manual redrawing (of the curve) may be eliminated.

Among the special time functions which may easily be realized technically and which have a greater technical importance at the same time, are the sine-function and the constant-plus-sine-function. They may be produced by a device designed, as Illustration (3) shows, by replacing the cylinder by a cam disk. The second possibility is to replace potentiometer 1 by a two-dimensional electrolytic tank with homogeneous field in which a sonde moves around on a circular path.

The next equation to be considered is the equation

12) $\ddot{x} = f_2\ (x).$

First, the function $f_2\ (x)$ has to be produced in scannable form. The magnitude x appears on the recording device as a mechanical impact which permits the registration of the function $f_2\ (x)$ if a contact slide or a sonde are coupled to the registering style, provided that the voltage pattern along the resistance or along the unidimensional electrolytic tank corresponds to the function $f_2\ (x)$. The desired voltage distribution is obtained by a method which is sketched in Illustration (4).

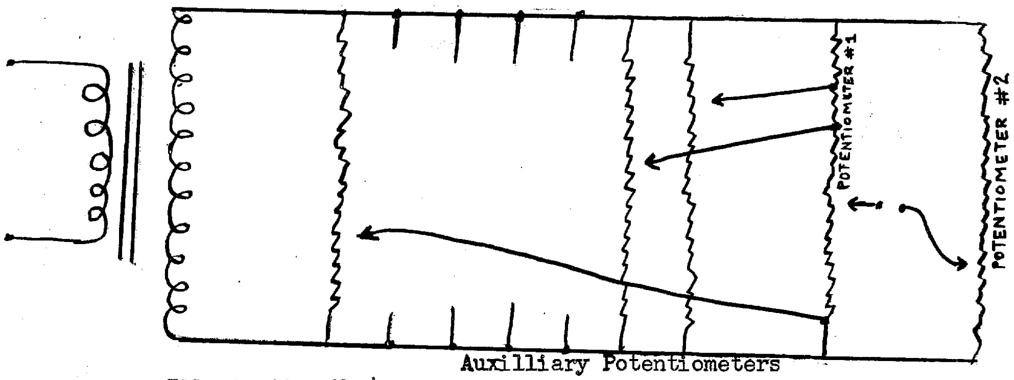

Illustration No.4

Adjustment of a certain voltage pattern along a potentiometer.

The potentiometer 1, which should be relatively highly-resistive, is supplied with current from less resistive auxiliary potentiometers at numerous points. The adjustment of the function is accomplished by adjusting the auxiliary potentiometers. Potentiometer 2 serves for re-adjustment of the zero point of the function value. If a unidimensional electrolytic tank is utilized in the place of potentiometer 1, the voltage supply is accomplished from the auxiliary potentiometers to the knife-shaped auxiliary sondes which are reaching a little bit below the water surface, thus permitting the scanning sonde to pass over without touching them. In many cases it is even possible to obtain a given function in an electrolytic tank along a straight line without auxiliary sondes and auxiliary potentiometers by modeling the environment of the scanning line along its breadth and/or depth accordingly. Thereby, one utilizes the known potential distributions between certain types of electrodes. Regarding the adjustment of the initial conditions, the same statements apply as in the first example.

The following examples are pertaining to the most important combination of $f_1(t)$ and $f_2(x)$. For the solution of equation

13) $$\ddot{x} = f_1(t) + f_2(x)$$

the voltages corresponding to the two functions of the right side, both of which are individually produced in the manner described in the previous example, must be added. For this purpose, the two voltages are conducted to the control grids of two pentodes whose anodes are interconnected, and to a common, relatively little resistive external resistance (approx. 10k). The total function will then appear on this resistance, the solution of the equation

14) $$\ddot{x} = f_1(t) = f_2(x)$$

is achieved by supplying the realization device for function $f_2(x)$ according to Illustration (4) not with a constant, alternating voltage, but with a $f_1(t)$-modulated voltage by means of the device according to Illustration (3). An appropriate amplifier may be interposed between the exit of the device according to Illustration (3) and the entry transformer to the device according to Illustration (4).

For the execution of subtractions and divisions, the models may either previously be adjusted as negative, e.g., reciprocating models, or an addition with 180° - reversed phase is executed for subtraction, and for a division, the enumerator and denominator functions are adjusted in a common automaticvolume-control-amplifier in such a manner that the denominator equals 1. Such a common regulation of enumerator and denominator is, mathematically speaking, the same thing as the multiplication (extension) of a fraction by the reciprocal value of the denominator so that the multiplied enumerator becomes equal to the value of the fraction. In order to be able to separate enumerator and denominator after the regulation process, the denominator function is previously transposed into a higher frequency range.

The solution of equations in which $f_3(\dot{x})$ occurs becomes relatively simple in

15) $$f_3(\dot{x}) = a \cdot \dot{x},$$

whereby a is a constant. The magnitude $\dot{x}$ is furnished by the tacho-generator, and the factor a is obtained through voltage division or voltage amplification. Also, combined equations with modifier 15 are easily soluble. Therefore, the method of solution of the equation

16) $$\ddot{x} = f_1(t) + f_2(x) + a \cdot x$$

distinguishes itself from equation (13) only by the fact that a further grid-voltage $a \cdot x$ is added to the already present addition pentodar.

However, if $f_3(\dot{x})$ is a more general function which has first to be produced by a scannable model, a further auxiliary device becomes necessary which distinguishes itself greatly from the models discussed so far.

In order to render the scanning of the function $f_3(\dot{x})$ possible, the contact slide or the sonde, as the case may be, must be moved proportionally to $\dot{x}$. As simple as this problem may sound, its solution presents nevertheless great difficulties provided that a high degree of accuracy is to be achieved. The most appropriate method is the utilization of a special error-controlled adjusting motor. The solution becomes easier by utilizing a two-phase induction motor. A potentiometer with a linear voltage distribution, whose contact slide is activated by the adjusting motor, is supplied with an alternating voltage whose frequency and phase are equivalent to the voltage generated by the tacho-generator. As shown by Illustration (4) a potentiometer 2 is located parallel to the potentiometer (cf. Illustration 5) for the purpose of zero adjustment. The voltage generated by the tacho-generator and a transformer are switched into the circuit between the two (contact) slides.

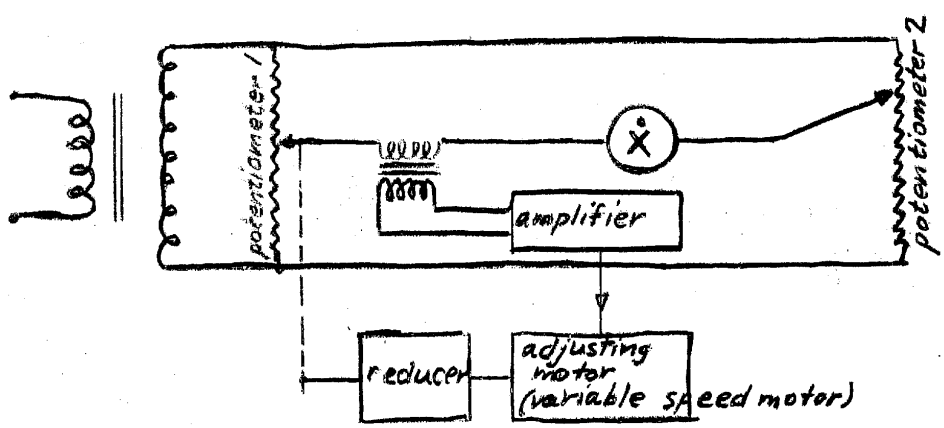

Illustration 5. Diagram of device for obtaining a deflection (of recorder) proportional to voltage $\dot{x}$.

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If the (contact) slide of the potentiometer 1 happens to be in the position in which a voltage of the magnitude results against the slide of potentiometer 2, no current flows through the transformer. The subsequent amplifier does not receive a control voltage and the adjusting motor does not generate any driving force. Inasmuch as the potentiometer 1 has a linear potential distribution, the distance of the (contact) slide from the zero position, as determined by potentiometer 2, is proportional to $\dot{x}$; the deflection from zero position is proportional to voltage $\dot{x}$. If the slide has a position slightly deviating from the desired value, however, or if the voltage $\dot{x}$ changes, in either case a current will flow through the transformer whose amplitude is proportional to the error and whose phase depends upon the sense of direction of the deflection. The adjusting motor receives a voltage via the amplifier which is driving it in the sense of correction toward the desired value until the fault-voltage disappears.

If the contact slide of another potentiometer is mechanically connected to the contact slide of potentiometer 1, and if a voltage pattern corresponding to the function f_3, according to the method as exemplified by Illustration 4, is adjusted, the voltage f_3 (x) may then be obtained by scanning. Thus, all the problems of the type of the equation (8) have become soluble.

If the restricting presupposition of equation (7) is to be dropped, and if the general function $f_o(\ddot{x})$ is to be introduced on the left side, the new component problem of displaying $\ddot{x}$ separately arises because $\ddot{x}$ is indispensable for the propulsion of the integrator aggregate. The solution of this component problem has a certain similarity with the display of the function $f_3(\dot{x})$, just discussed.

In order to realize the magnitude $\dot{x}$ in the form of an electrical voltage in which it is being utilized, it is appropriate to proceed via the detour of realizing it first as a mechanical magnitude - a distance - and to subsequently obtain from this mechanical magnitude the desired electrical magnitude. The solution is again achieved by means of a special error-controlled adjusting motor. The required auxiliary device has all the features of the diagram of Illustration 5. The only differences are the facts that the potentiometer 1 is replaced by one on which the voltage distribution may be adjusted according to the function f_o - i.e., the auxiliary potentiometers according to Illustration 4 are also required - and that the voltage corresponding to the right side replaces the voltage $\dot{x}$.

While the system is in operation, the contact slide has the distance $\ddot{x}$ from the zero position. If a mechanical connection between the contact slide of potentiometer 1 and the contact slide of another potentiometer on which a linear voltage distribution prevails is established, it becomes possible to tap the voltage $\ddot{x}$ between the contact slide of this potentiometer and the corresponding zero position adjuster slide. Thereby all the prerequisites for the solution of the equation

17) $$f_o\ (\ddot{x}) = f_1\ (t) \bigcirc\ f_2\ (x) \bigcirc\ f_3\ (\dot{x})$$

are fulfilled.

No additional problem is created, if, as already mentioned in chapter II, the left side is multiplied with a function of t, x, or $\dot{x}$, because this case only demands that the potentiometer with the voltage distribution, according to the function f_o, be supplied with a modulated alternating voltage instead of with an alternating voltage of constant amplitude.

From the mathematical viewpoint, there will, therefore, be no new aspects in the adjustment of the initial conditions. However, if one considers that the adjusting mechanism for $\ddot{x}$ has an inertia effect, however small it may be, it is nevertheless desirable to see to it that also the adjusting mechanism shows approximately the correct value already at the time the system is switched on. This value may be obtained by estimate or by a previous experiment.

V. The solution of a non-linear differential equation of 2nd degree whose single functions are depending on two magnitudes.

The characteristic feature in the solution of this type of equation is the utilization of two-dimensional models. In the one-dimensional models the coordinate application of the slide resistance and the one-dimensional tank are the basic technical means for the (mathematical) display of the single functions. Both (slide resistance and tank) have their advantages as well as disadvantages. The resistance may be used in any position, even on vehicles, whereas the electrolytic tank must be mounted in a vibration-proof arrangement and kept horizontally aligned. In the case of the electrolytic tank, on the other hand, the contact is particularly reliable; it may be adjusted with a minimum power loss and has no dead run (play) which, in turn, may easily occur in the case of a slide resistance. The same principles are basically applicable to the two-dimensional models; however, the two-dimensional tank has greater possibilities because there are no resistances available which would satisfy the technical requirements. As a rule, auxiliary sondes in the form of pins will be introduced through the bottom of the tank for the adjustment of the function. In many cases, it may be possible also to reach one's objective through echelonment in depth or through special electrode shapes. As a "dry model" in the two-dimensional scope, the application of a resistance foil or a resistive lining may be feasible which could be scanned on one side, whereas the contacts for the adjustment of potentials would be located on the opposite side. Foil and lining may also be cylindrical, which would result in a particularly convenient scanning and over-all arrangement in a similar manner as in the case of utilizing a variable resistor (rheostat) in the place of a slide resistance for one-dimensional models. In the subsequent discussion of the two-dimensional models, we shall speak about the electrolytic tank. All pertinent concepts may be adapted to the dry model or accordingly modified for cylindrical devices.

The plane motion of the sonde may be effectuated by a device comparable to a bridge crane. In two parallel walls of the tank are two guide rails on which the bridge is movable across the entire tank. The mobile sonde carrier is mounted on the bridge in vertical direction to the bridge movement, in similar manner as a crane carriage. In the operational application, the motion has to be carried out in such a manner that the distances from the zero positions correspond in any direction to the assigned magnitude t, x, or $\dot{x}$. "Selsyn" (sic) systems are particularly suited for the transmission of the forces necessary for the movements of the bridge and sonde carrier. These (Selsyn) systems are the electromechanical systems for telemetric transmission of angles of rotation; they require less electrical equipment than the servomechanism exemplified by Illustration 2. On the other hand, the moment of rotation cannot be amplified either. The mechanical separation from the integrator unit is desirable in order to permit the mechanically independent installation and adjustment of each unit.

After the discussion of the problems in the previous chapter, there is little if anything to say regarding the execution of the arithmetical operation. It is most advisable to review the theories set forth, by applying them to particularly characteristical problems.

In the integration of the differential equation

$$\ddot{x} = f_1\ (t, x),$$

the direction assigned to the magnitude t is passed through on the model at a constant rate of speed, whereas the motion toward the other axle has to be transmitted by the integrator.

For the solution of differential equations of the type

17) $\ddot{x} = f_2\ (x, \dot{x})$ and

$\ddot{x} = f_3\ (t, \dot{x}),$

as well as

18) $f_0\ (\ddot{x}, f) = f\bigcirc . \ldots .$

$f_0\ (\ddot{x}, x) = f\bigcirc . \ldots ,$

one-dimensional auxiliary tanks are required - in the case of equation (17) in order to execute the motion in the $\dot{x}$ - direction proportionally to the voltage, and in equation (18) in order to realize the magnitude $\ddot{x}$ as a voltage. The switching characteristics are the same as in one-dimensional models. It is most appropriate to arrange the one-dimensional auxiliary tanks in such a manner as to support the sonde of the auxiliary tank by a bracket arm fixed to the bridge.. A corresponding assignment of coordinates at the main tank has to be observed.

For the solution of an equation of the type of

19) $f_0\ (\ddot{x}, \dot{x}) = f\bigcirc . \ldots ,$

two-dimensional auxiliary tanks are required; their individual propulsion problems may be solved either mechanically or electro-mechanically. New aspects arise in the formation of right sides in connection with several functions. Also the adjustment of the initial conditions is executed accordingly to the previously developed scheme.

VI. The solution of systems of non-linear differential equations of 2nd degree and the basic principle of the electron path recorder.

For the solution of systems of non-linear differential equations of 2nd degree, as many integrator aggregates as there are simultaneous differential equations in the system are required to operate simultaneously. This sentence already contains in itself all essential new features of the calculating machine for systems of differential equations. The formation of the right sides may simply become somewhat more involved and more complicated, but there are no new concepts connected with this sort of operation. The fact that functions of the other variables and of their quotients appear now also on the right side involves no new principles. The number of initial conditions to be adjusted is, as a matter of course, proportional to the number of simultaneously appearing equations.

Several peculiar problems arise in the dynamic electron path recorder. The most generally formulated system of differential equations for electron motion on a plane with the coordinates x, y reads

20) $$\ddot{x} = \frac{e}{m}\ (E_x - H_2\dot{y})$$

$$y = \frac{e}{m}\ (E_y\ \ H_z\dot{x}).$$

$\frac{e}{m}$ is the relationship between charge and mass of the electron; E_x and E_y are the components of the intensity of the electric field; generally speaking, therefore, they are non-linear functions of x and y, and in the case of very high frequencies (VHF) also functions of time. H_z is the intensity of the magnetic field which may only occur vertically to the x, y plane. It is an implicit requirement that E_z disappear already at the beginning of the operation.

If this problem were handed to a mathematician who is familiar with the concepts so far discussed, as well as with the general calculating machine, and provided that he did not introduce some new physical concepts, he would pronounce the problem soluble, if the occurring dependence of E_x and E_y from t may be executed by one of the basic arithmetical operations through associating these magnitudes with a time function. The latter presupposition may practically always be realized. The dependence from the time (function) may be executed through a modulation according to equation (14). The mathematician would raise the functions E_x $E_x (x,y)$ and E_y $E_y (x,y)$. They would probably be obtained through measurements of the potential field and subsequent calculations based on these measurements. The mathematician would set up two of his two-dimensional tanks, prepare the time function and thus let the mathematical operation take its course.

The physicist considering the problem from his viewpoint may discover another method which would greatly reduce the work necessary for the solution. The functions E_x and E_y may be obtained directly by scanning of the electrolytic tank for measurement of the potential field, and, consequently, this tank may be utilized as basis for the realization of the right side (of the equation), thus rendering all auxiliary tanks superfluous. Instead of the original three tanks, two of which would have been rather complex because of the required settings of the potential fields, there remains only one tank. In this uncomplicated tank, the entire operation of modelling may be performed simply by introducing the electrode model into the arrangement under investigation. The technical trick with which the physicist renders the solution of the problem so simple is the application of a quadruple sonde in the place of the conventional simple sondes. By means of a pair of sondes, the potential difference may be measured in one direction, e.g., an approximation toward the component of field intensity in the considered direction. Two sonde pairs are combined in the quadruple sonde, one each in the x and the y direction.

Inasmuch as a tank with a systematic electrode arrangement serves as a model, it is convenient to display the motion also in the plane in which the electrodes are displayed. This is simply accomplished by recording of the movements of the sonde carrier. In order to registrate the time factor, a point system with temporally equidistant points is plotted.

Any further principal problems may be solved from the general theory. Temporal modulations of the field intensity may be executed through a modulation of the electrode voltage in the manner already mentioned. All the other problems of development are of a technical nature and have to be discussed from the viewpoint of the general techniques of experimentation and instrumentation.